Solution Checker for the Dual Master Route Ratio LP

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The Python script `verifier_master_route_ratio.ipynb` can read in a solution to the (Dual-Master-Route-LP), check its feasibility, and output the objective value. The script can be opened and run by JupyterLab or jupyter-notebook (see `https://jupyter.org`).

The script works with upper and lower bounds for some constraints, thus the solution is not necessarily infeasible if the script reports that some constraint is not met. However, by the choice of the bounds, we can guarantee that the solution is feasible if the script says so.

If the provided solution is infeasible, the script reports the first constraint that is not fulfilled; if the script identifies the solution as feasible, it outputs the objective value as well as an upper bound on the inverse of the objective value. (The inverse is an upper bound on the master route ratio.)

Note that we set the constants that are involved to $\beta = \frac{1}{400}$, $a = 199$ and $N = 4000$.

$$\max \sum_{i=1}^{N} i\beta^2 \cdot y_i \qquad \text{(Dual-Master-Route-LP)}$$

$$\text{subject to} \quad y_i + \sum_{j=i-a}^{i} z_j + \mathbb{1}_{i=1} \sum_{j=2}^{N} j \cdot x_j - \mathbb{1}_{2 \le i \le N} \cdot x_i \le e^{-(i+\frac{1}{2})\beta} \quad \text{for } i = 0, \dots, N \tag{1}$$

$$2\beta \cdot y_i \le v_{j,i} + w_{j,i} \quad \text{for } i = 1, \dots, N \text{ and } j = 1, \dots, \left\lceil \frac{h_i + i + 1}{a} \right\rceil \tag{2}$$

$$\sum_{i=1}^{N} \sum_{j=1}^{\left\lceil \frac{h_i+i+1}{a} \right\rceil} \left(\mathbb{1}_{k=(j-1)a-h_i} \cdot v_{j,i} + \mathbb{1}_{k=i-ja+h_i} \cdot w_{j,i} \right) = z_k \quad \text{for } k = -a, \dots, N \tag{3}$$

$$x, y, v, w \ge 0 \ . \tag{4}$$

1 Input Format

The solution is given in the file `sol_mrr_lp.txt`. We encode it using only integers so that the Python script can do exact arithmetic operations instead of floating point operations with unpredictable rounding errors. Since the solution to the LP itself is, of course, not integral, we scale all values of the variables by a factor of 10^{10}.

The first line consists of the letter `y`, followed by lines that each contain two integers i and c, defining $y_i = c \cdot 10^{-10}$. Next are analogous blocks for z and x.

Then there is a line with the letter `v`, followed by lines that each contain three integers i, j, and c, defining $v_{i,j} = c \cdot 10^{-10}$. Finally a block starting with the line `w` defines the w-variables in an analogous way as the v-variables.

The value of a variable for which the combination of its indices does not appear in the file is implicitly set to 0.

Example

```
y
401 16902662
402 27559454
⋮
z
-199 18520635
-198 39322560
⋮
x
106 1265589
107 1624539
⋮
v
1 1 1
1 2 1
⋮
w
2 401 84555
2 402 137867
⋮
21 4000 2266
```

2 Checking the Constraints

2.1 Checking (1)

We work with lower bounds ℓ_i for $e^{-(i+\frac{1}{2})\beta} \cdot 10^{10}$ as follows: For $i = 0$, we have

$$e^{-\frac{1}{2}\cdot\beta} \cdot 10^{10} > 9987507809 =: \ell_0.$$

We recursively define

$$\ell_{i+1} := \lfloor \ell_i \cdot 9975031223 \cdot 10^{-10} \rfloor.$$

Then $\ell_i \leq e^{-(i+\frac{1}{2})\beta} \cdot 10^{10}$ for all $i \geq 0$ since $e^{-\beta} > 9975031223 \cdot 10^{-10}$.

We then can check all inequalities of type (1) with integer operations only by replacing the right hand side by ℓ_i and multiplying the left hand side with 10^{10}. (Note that this multiplication of the left hand side is done implicitly since we maintain all variable values scaled by a factor of 10^{10}.)

2.2 Checking (2)

We can check these inequalities by verifying that

$$10^{10} \cdot 2 \cdot y_i \leq \frac{1}{\beta} \cdot (v_{j,i} + w_{j,i}) \cdot 10^{10}.$$

2.3 Checking (3)

Multiplying both sides of the equation with 10^{10} gives us an integer comparison.

2.4 Checking (4)

This is straightforward.

3 Computing the Objective Value

The inverse of the objective value gives us an upper bound on the master route ratio. So we are interested in the value of $\frac{\beta^{-2}}{\sum_{i=1}^{N} i \cdot y_i}$. Hence

$$\left(\left\lfloor \frac{\beta^{-2} \cdot 10^4 \cdot 10^{10}}{\sum_{i=1}^{N} i \cdot y_i \cdot 10^{10}} \right\rfloor + 1 \right) \cdot 10^{-4}$$

is an upper bound on the master route ratio.

Running the script on the provided input yields as output an upper bound of $25839 \cdot 10^{-4}$.